TALASIĆI (WAVELETS)
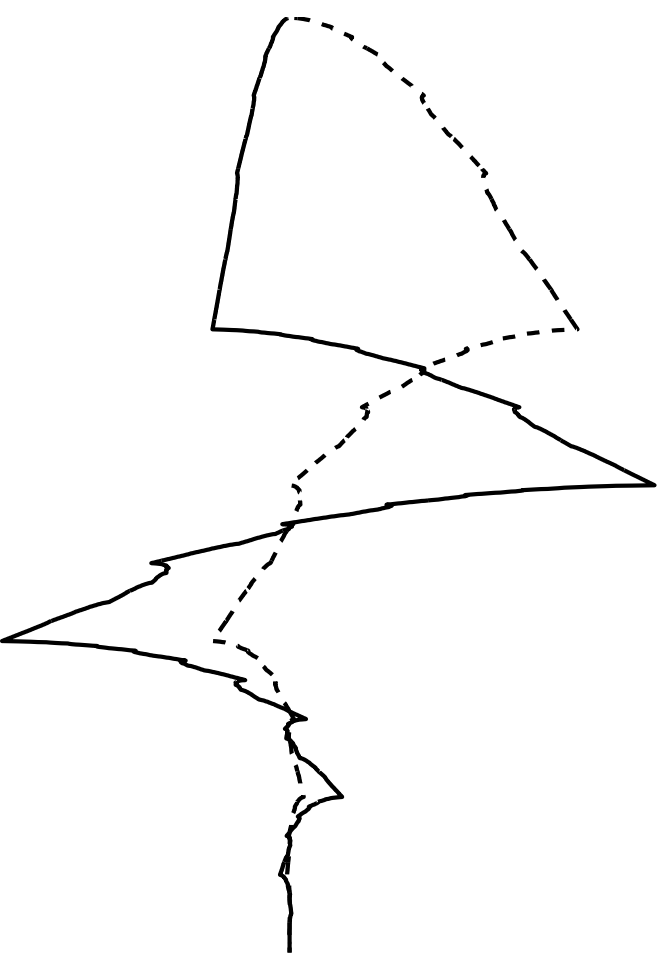
1. Transformacija

2. Multirezolucija

3. Konstrukcija

4. Filtar

5. Osobine

6. Piramidalni algoritam

7. Primeri i primene

2007

Fourier-ova analiza (Joseph Fourier, 1807)

$$f(x) \approx \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx) = \sum_{k=-\infty}^{\infty} c_k e^{\imath kx}$$
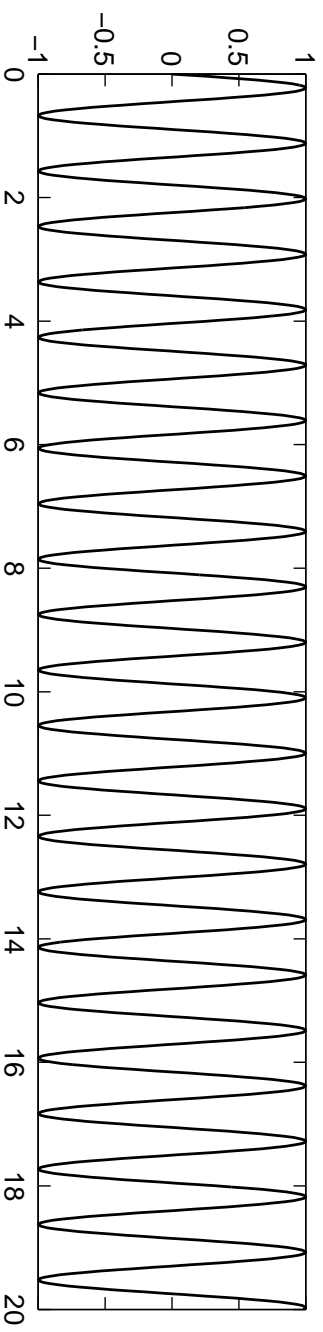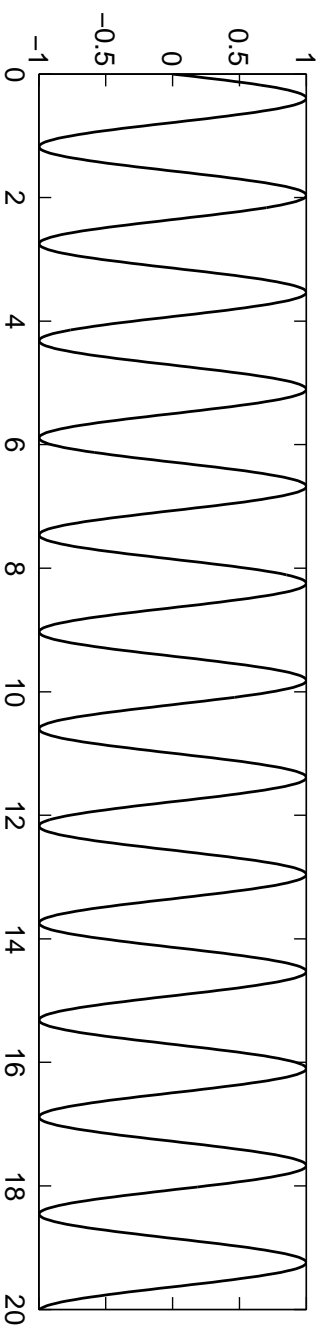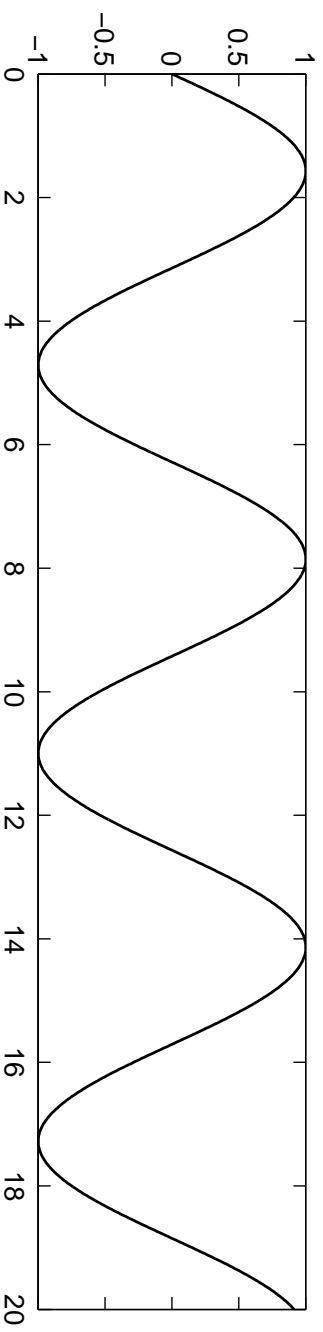
$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx \, dx$$
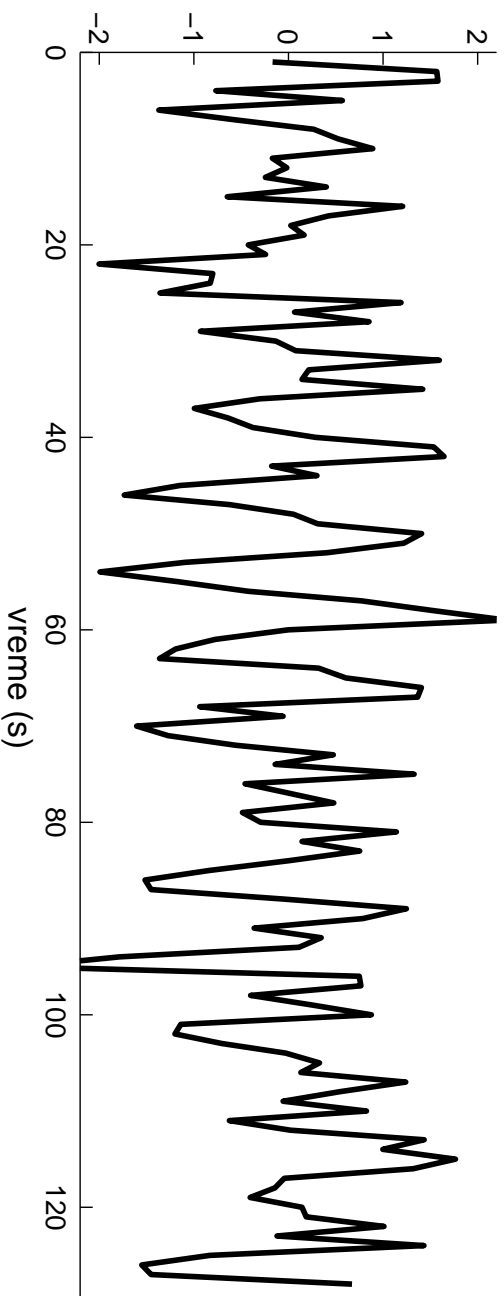
$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx \, dx \qquad c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-\imath kx} \, dx$$

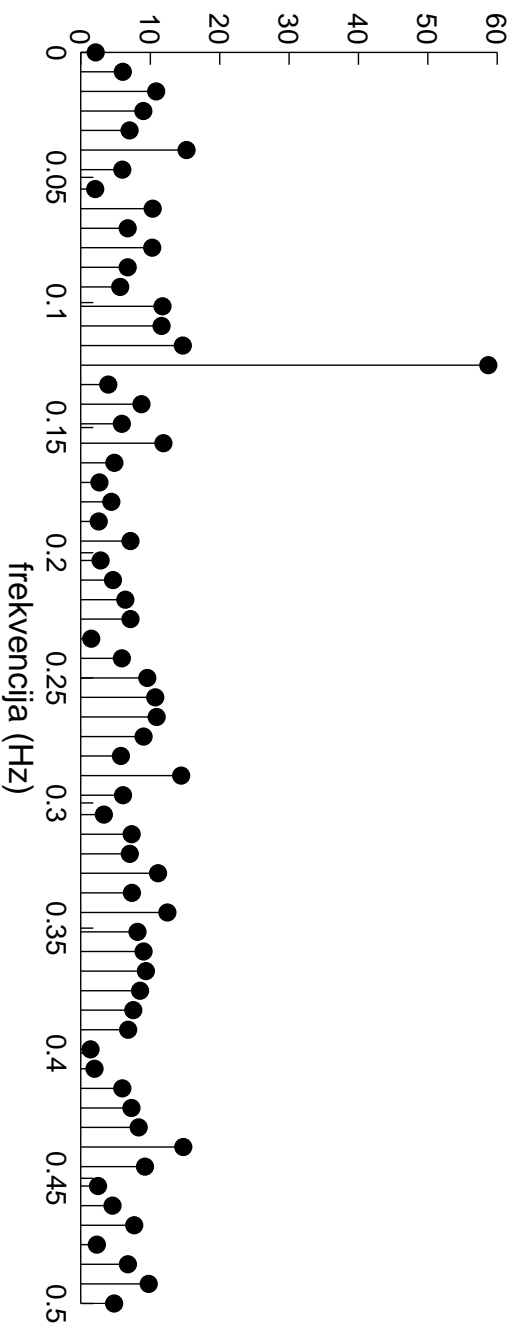Parseval-ova jednakost – energija funkcije f

$$\|f\|^2 = \int_{-\pi}^{\pi} |f(x)|^2 \, dx = \sum_{k=0}^{\infty} (|a_k|^2 + |b_k|^2) = \sum_{k=-\infty}^{\infty} |c_k|^2$$

Harmonici $1, \sin x, \cos x, \sin 2x, \cos 2x, \ldots, \sin nx, \cos nx, \ldots$
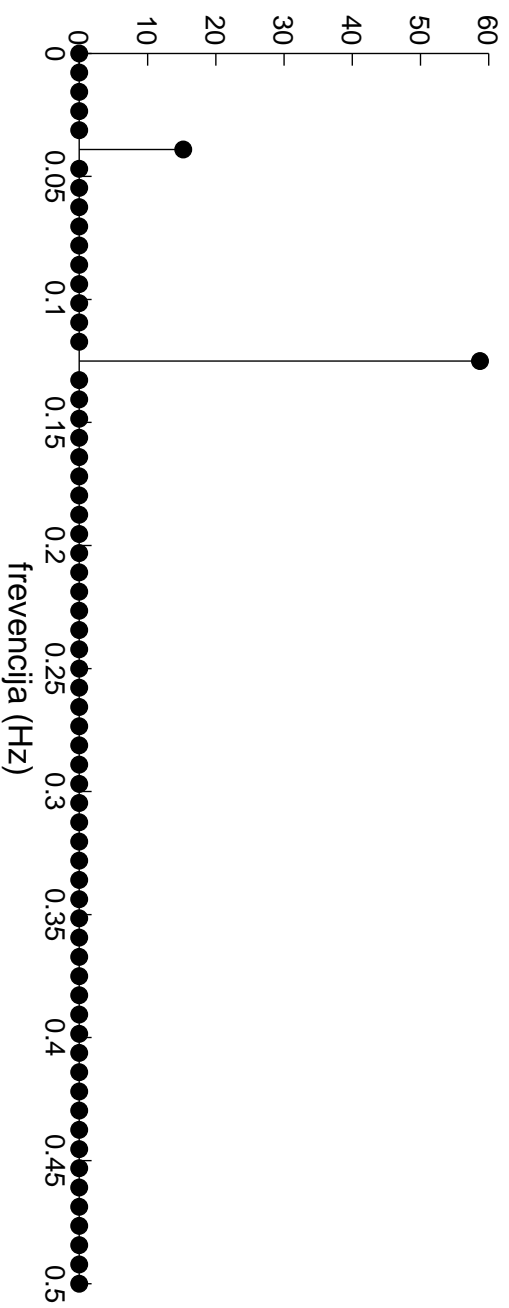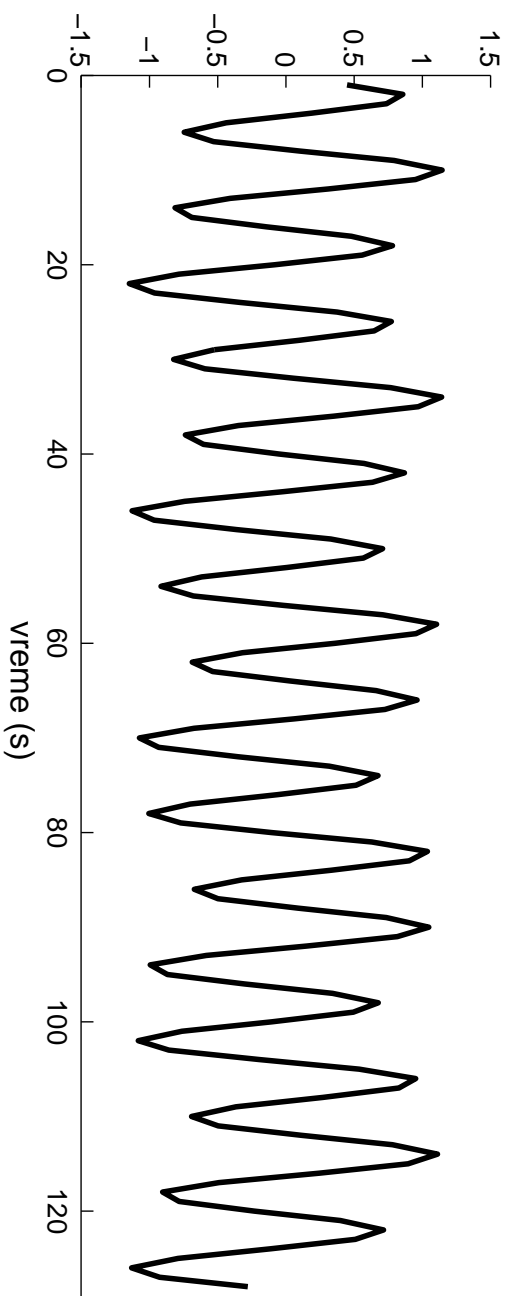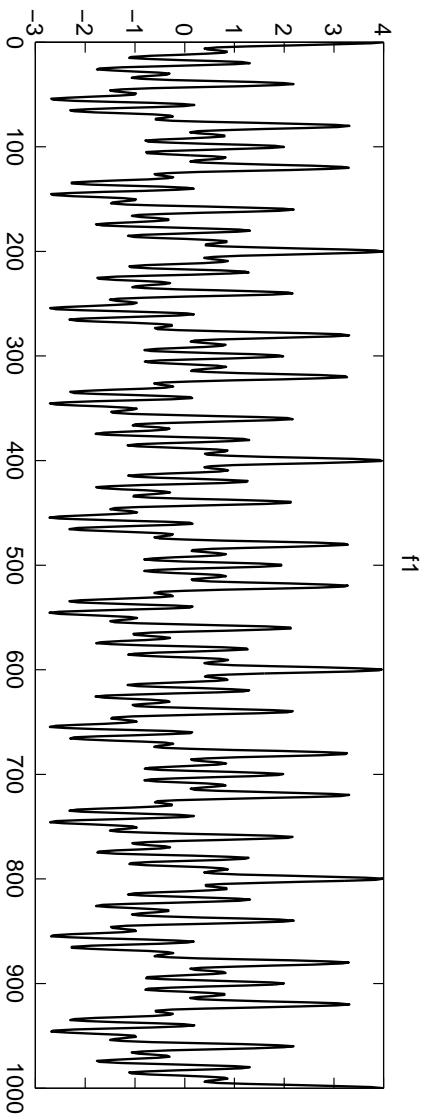
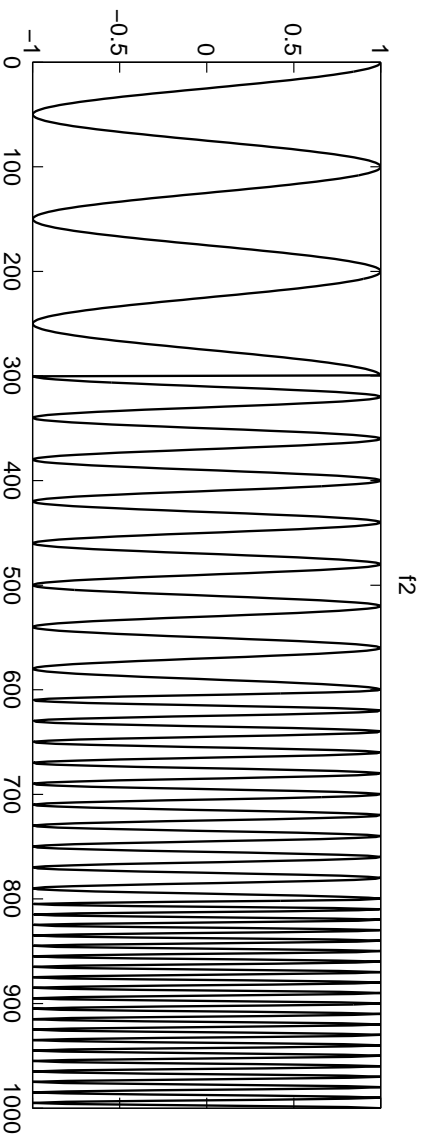Vremenski domen signala

Frekvencijski domen signala

Kompresija signala u frekvencijskom domenu

vreme (s)

frevencija (Hz)

f1

f2

$$\cos\left(2\pi * 10 * x\right)$$
$$+\cos\left(2\pi * 25 * x\right)$$
$$+\cos\left(2\pi * 50 * x\right)$$
$$+\cos\left(2\pi * 100 * x\right)$$

$$\left.\begin{array}{ll} \cos\left(2\pi * 10 * x\right), & [0, 300] \\ \cos\left(2\pi * 25 * x\right), & [300, 600] \\ \cos\left(2\pi * 50 * x\right), & [600, 800] \\ \cos\left(2\pi * 100 * x\right), & [800, 1000] \end{array}\right\}$$

Fourier-ov spektar
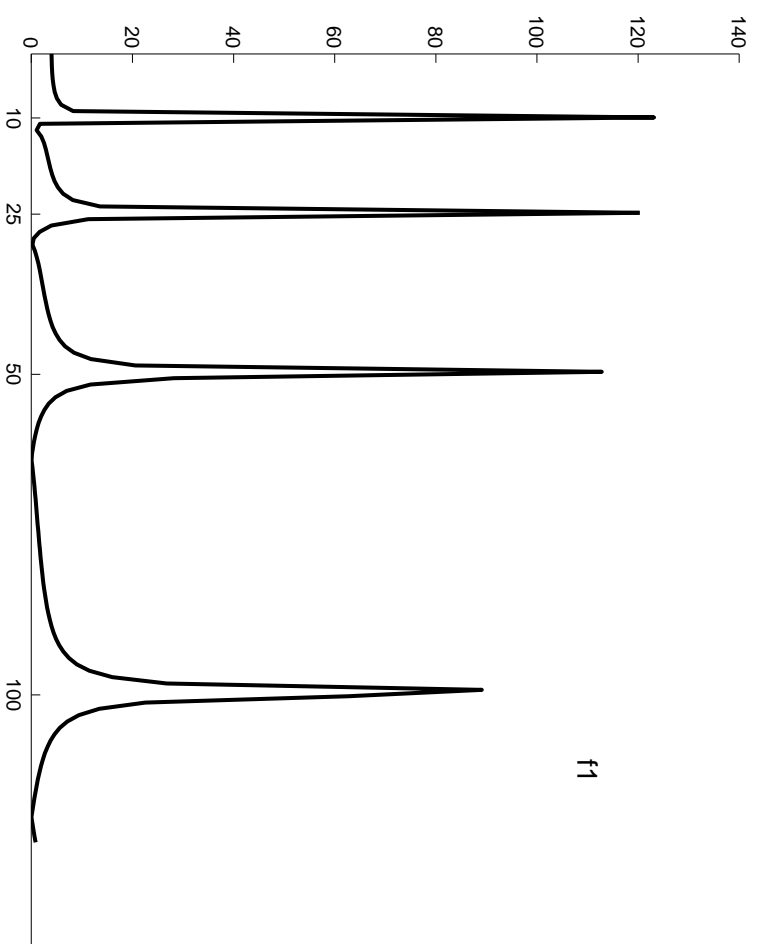
stacionarnog signala,
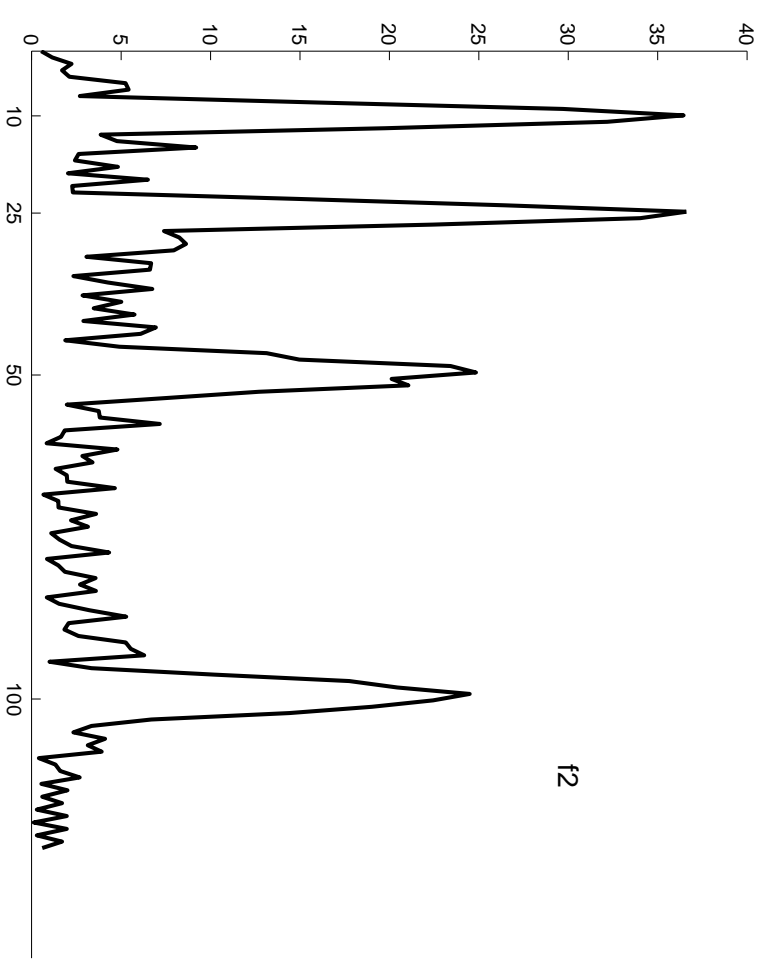
nestacionarnog signala

f1

f2

Transformacija

Fourier-ova

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-\imath x \omega} dx \qquad \left(c_k = \int_{-\pi}^{\pi} f(x) e^{-\imath k x} dx \right)$$

Kratkotrajna Fourier-ova

$$STFT_f(\omega, \tau) = \hat{f}(\omega), \quad x \in [\tau, \tau + 1]$$
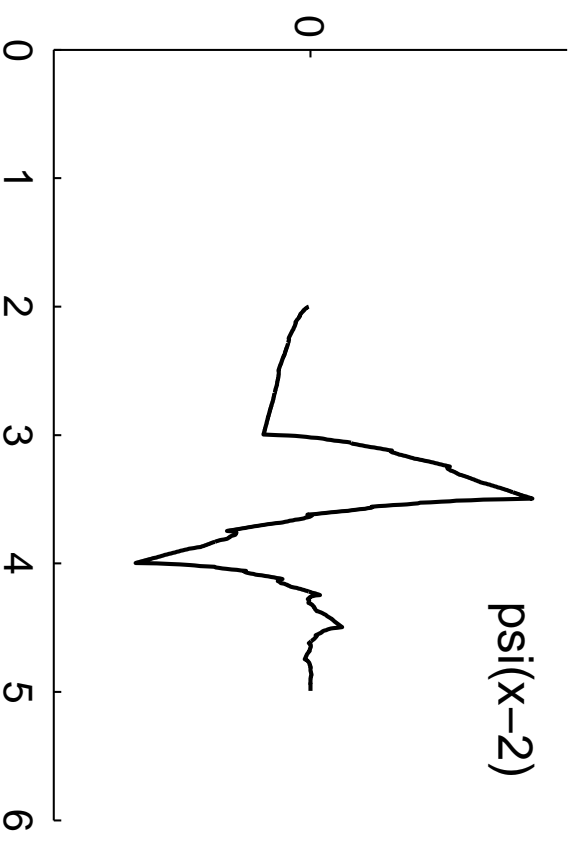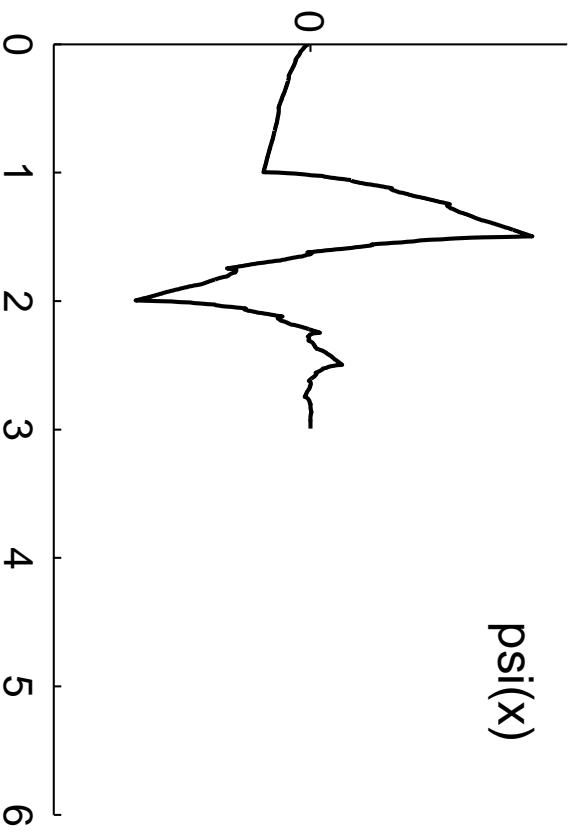
Fourier-ova transformacija funkcije $f(x) W(x - \tau)$

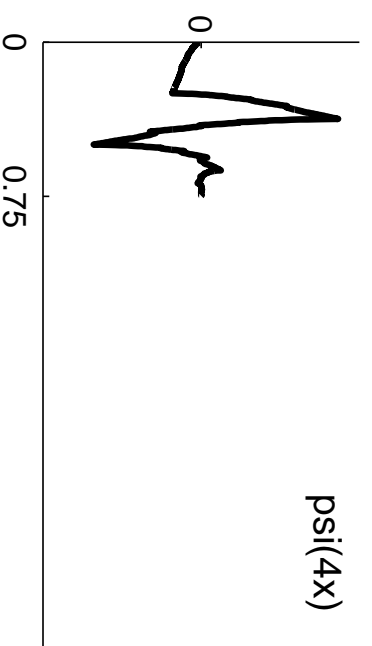Prozorska funkcija $\qquad W(x) = \begin{cases} 1, & x \in [0, 1) \\ 0, & x \notin [0, 1) \end{cases}$

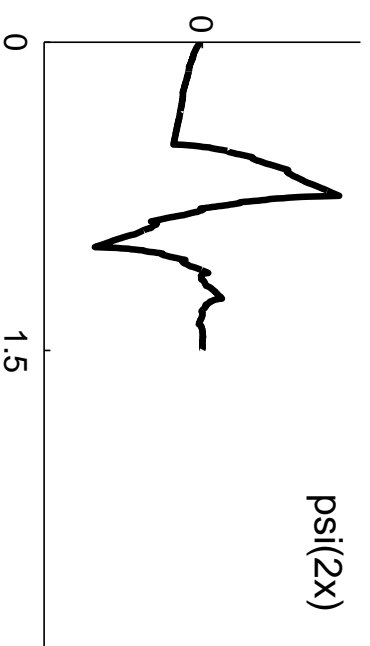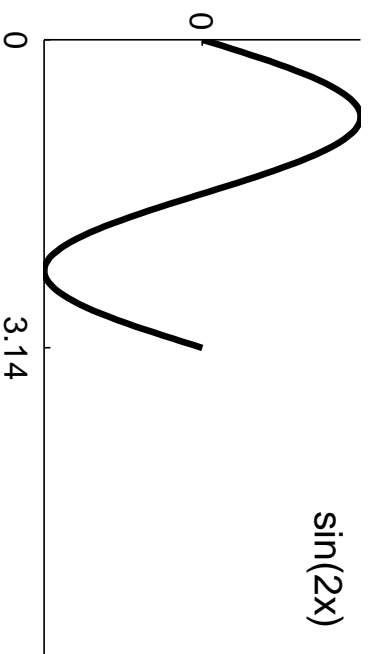## Talasićima

$$WT_f(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x) \, \overline{\psi}(\frac{x - b}{a}) dx$$

Talasić — oscilatorna funkcija sa kompaktnim nosačem

Translacija talasića — vremenska rezolucija (parametar b)

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}}\,\psi\!\left(\frac{x-b}{a}\right)$$

psi(x)

psi(x−2)

sin(x)

sin(2x)

sin(4x)

psi(x)

psi(2x)

psi(4x)
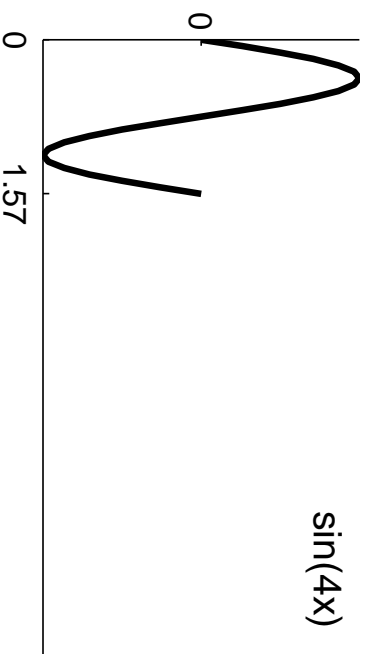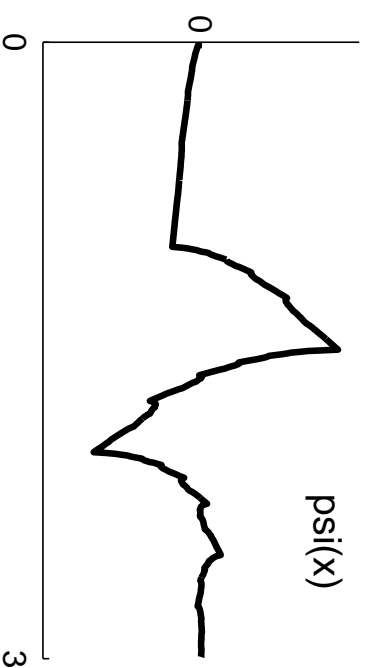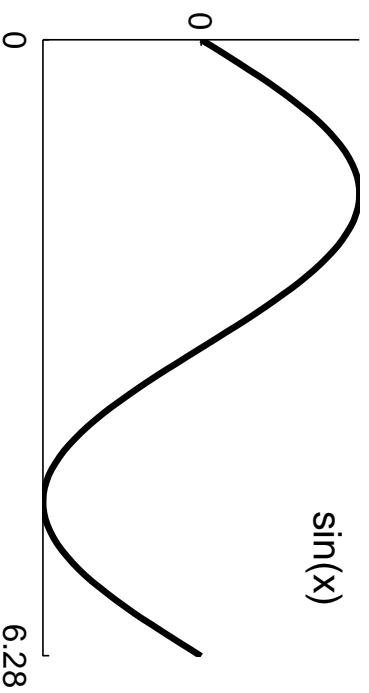
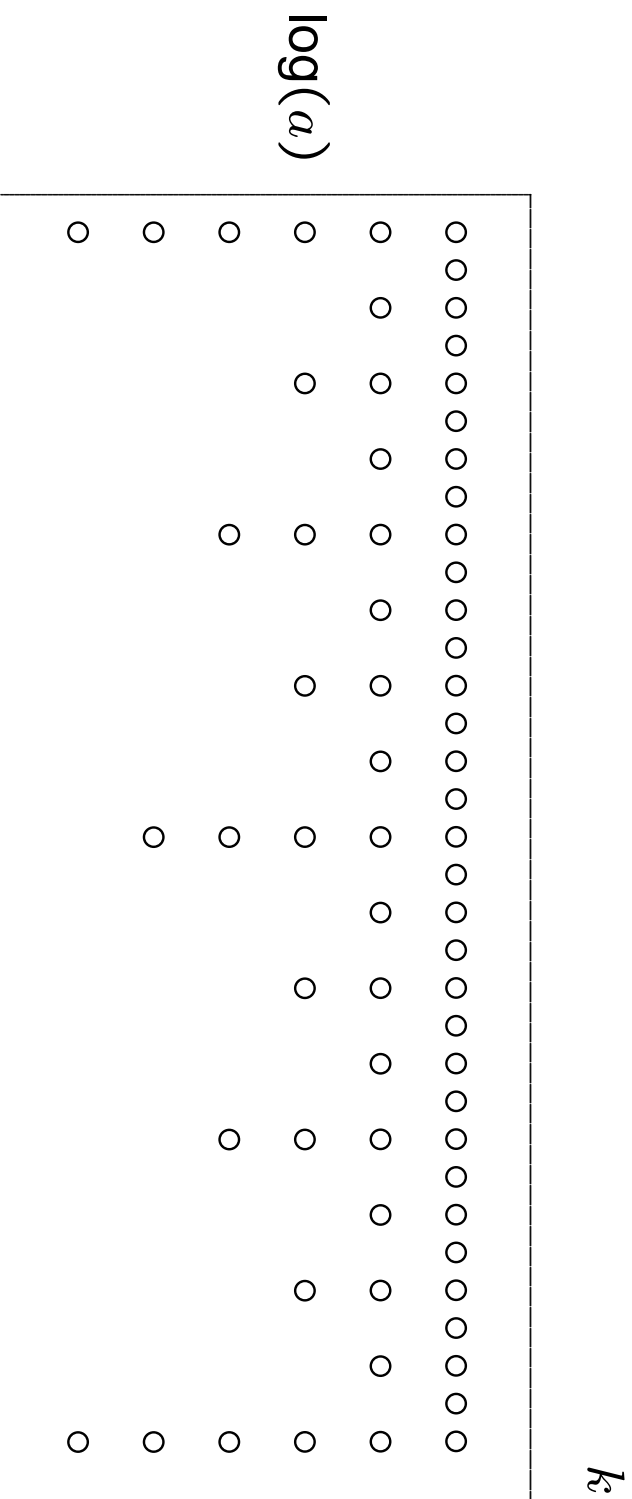Dilatacija
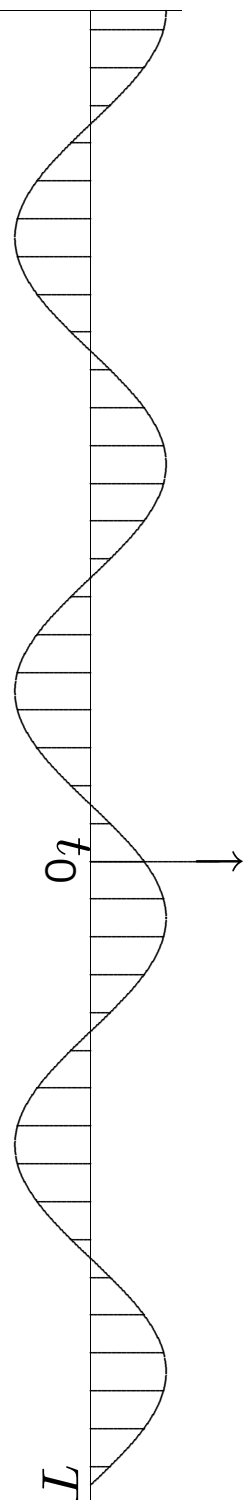sinusoide
i talasića
–
frekven-
cijska
rezolucija
(param. a)

Diskretni talasići

$$\psi_{jk}(x) = 2^{-j/2}\psi(2^{-j}x - k), \qquad \psi_{jk}(x) \neq 0, \quad x \in [2^j k, 2^j(k+1)].$$

$$a = 2^j, \quad b = k2^j, \quad k, j \in \mathbb{Z}$$

$\log(a)$

k

Primer

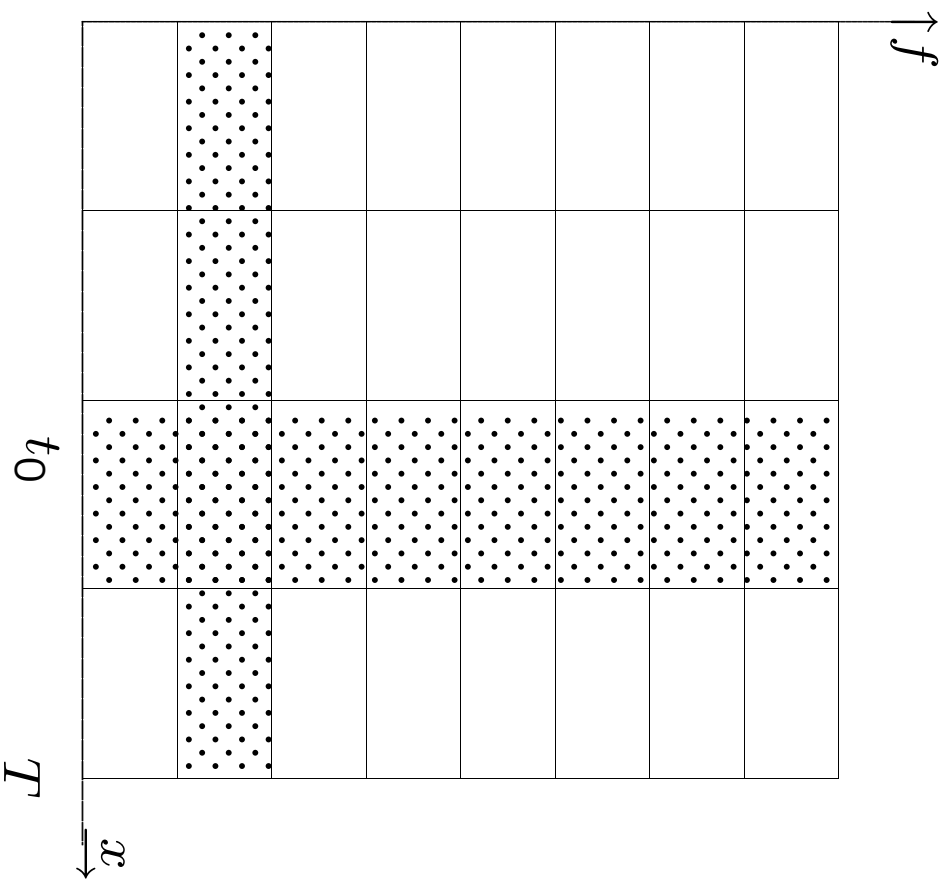$$s(x) = \sin x + \delta(x - t_0)$$

$$\approx \sin x + \frac{1}{2\pi}\left(1 + 2\cos(x - t_0) + 2\cos 2(x - t_0) + \cdots\right)$$

STFT $_f$

WT $_f$